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Subadditive measure on projectors of von neumann algebra

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ABSTRACT

In the paper subadditive measure on the lattice of orthogonal projectors of von Neumann algebra is considered. The basic peoperties of the subadditive measure are established and proved.

Keywords: The lattice of projectors, equivalent projectors, subadditive measure, finite measure, affiliated with the algebra, spectral decomposition.

1. INTRODUCTION

Let us assume that H is a complex Gilbert space, given the algebra of all finite operators defined in \mathcal{B} (N) - N. Von Neumann's algebra is such a set of M that is a partial algebra of \mathcal{B} (N) (that is $a \in M$, if it is), if it is closed to the top operator of the unit, including the operator.

The commutant of M algebra is a set of M' such $a \in M$ that this set consists of all $b \in M$ that this

All orthogonal projectors in M von Neumann's algebra form a grid (logic). We define it ∇ through us. From now on we call the elements projectors.

Two projectors $p,q\in \nabla$ are called equivalents, $u\in M$ if any $u^*u=p$, $uu^*=q$. Specify the $p\sim q$ equivalent projectors. If p the projector is q equivalent to a part of the projector, then we use the designation p< q. Through p^\perp the projector 1-p. Through $p\vee q$ the projection grid and the upper upper limit (s) of the projector; and the lower bound (infimum).

For M von Neumann's algebra $p, q \in \nabla$ it is shown $p \vee q - q \sim p - p \wedge q$ in [1].

Definition 1. A subadditive dimension in M von Neumann's algebra $m: \nabla \to [0, \infty)$ is said to be expressed as follows:

- a) m(0) = 0; m(p) = 0 because of p=0;
- b) $p \le q$ because of $m(p) \le m(q)$;
- c) $p \sim q$ because of m(p)=m(q);
- $d) \qquad m(p \vee q) \leq m(p) + m(q)$
- e) $p_n \uparrow p$ because $m(p_n) \uparrow m(p)$ if the origin

the scale is called the limit, if $m(1) < \infty$ any. In relation to [1] above $p \lor q - q \sim p - p \land q$, we can substitute the condition $m(p \lor q) = m(p+q) \le m(p) + m(q)$ for projectors that are (d) $p \perp q$ instead of condition (d')

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Definition 2. The linear part space of the H Gilbert space $D \subset H$ (operator α , respectively defined in H Gilbert space) is called M if $u' \in M$ it is for the optional unit operator $u'(D) \subset D$ (as appropriate au' = u'a). In this case the $D\eta M$ marking (as appropriate $a\eta M$) is used.

For any closed, densely defined operator a, it is defined as a module $|a| = (a * a)^{\frac{1}{2}}$.

If
$$|a| = \int_{0}^{\infty} \lambda de_{\lambda}$$
 the spectral distribution of ([2]) is, then $a \eta M$ it is $e_{\lambda} \in \nabla$ necessary

and sufficient for α a M and for $u \in M$ that element in the polar distribution of operator $a = u \mid a \mid$.

The following statement provides the main features of the \mathcal{M} subadditional dimension.

Theorem. [5] For the subadditive M dimension, the following properties are applicable:

1.
$$p < q$$
 becouse $m(p) \le m(q)$.

$$2. m(\bigvee_{i+1}^{\infty} p_i) \leq \sum_{i-1}^{\infty} m(p_i)$$

3. If
$$p(H) \subset D(a)$$
, $||ap|| < \lambda$ so $m(e_{\lambda}^{\perp}) \leq m(p^{\perp})$, on this

$$\mid a \mid = \int\limits_{0}^{\infty} \lambda de_{\lambda}$$
 , $a\eta M$ densely defined operator;

4.
$$m(p) < m(q)$$
 because $p^{\perp} \wedge q \neq 0$;

5. The projector $q \in \nabla$, for any $\varepsilon > 0$, $m(p^{\perp}) \leq \varepsilon$ and $p \wedge q = 0$ if there is a condition satisfying $p \in \nabla$, then q = 0.

6. Projector $q_1,q_2\in \nabla$, for any $\varepsilon>0$, $m(p^\perp)\leq \varepsilon$ and $q_1\wedge p=q_2\wedge p$ if there is a satisfying condition $p\in \nabla$, then $q_1=q_2$.

Proof. Attribution 1 comes directly from the conditions of definition 1 (b) and (c).

2 properties are derived from conditions 1 (d) and (e) of Definition 1.

3. any n is appropriate for a natural number $e_{\lambda + \frac{1}{n}}^{\perp} < p^{\perp}$. (See [4]).

$$e_{\lambda+\frac{1}{n}}^{\perp} \uparrow e_{\lambda}^{\perp}$$
 it follows $m(e_{\lambda}^{\perp}) = \sup_{n} m(e_{\lambda+\frac{1}{n}}^{\perp})$ from the condition of definition 1 (e).

This follows from the properties of (a) $m(e_{\lambda}^{\perp}) \leq m(p^{\perp})$ of this theorem.

4.
$$q-q\wedge p^{\perp}\sim p-q^{\perp}\wedge p$$
 and becouse $p^{\perp}\wedge q=0$ and $q\sim p-q^{\perp}\wedge p\leq p$ is derived. From this $p^{\perp}\wedge q=0$ because $m(q)\leq m(p)$ is derived.

5. We own
$$q=q-p \land q \sim p \lor q-p \le p^{\perp}$$
. In this case $m(q) \le m(p^{\perp}) \le \mathcal{E}$. Since $\varepsilon > 0$ is optional and m its net value is $q=0$.

6. Checking that $(q_1-q_1\wedge q_2)\wedge p=0$ is not difficult. It $q_1-q_1\wedge q_2=0$ follows that this theorem is based on the property of (v). It can be shown that this is the case $q_2\leq q_1$.

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Example 4. For each m subaddit measure $\overline{m}: \nabla \to [0,1]$, we define:

$$\bar{m}(p) = \begin{cases} m(p), & if & m(p) \leq 1; \\ 1, & if & m(p) > 1 \end{cases}$$

It is not difficult to check $ar{m}$ whether a subadditional measure is abla .

Example 5. Let $\psi(\lambda)$, $\lambda \geq 0$ us assume, for example, that there is a decreasing and nonnegative function, that is, $\psi(\lambda) > 0$, $\psi(0) = 0$, semi-additive, that is $\lambda_1, \lambda_2 \geq 0$, for the left, continuous $\psi(\lambda_1 + \lambda_2) \leq \psi(\lambda_1) + \psi(\lambda_2)$. For any m subadditive dimension

$$m_{\psi}(p) = \psi(m(p)), \quad p \in \nabla$$

The function $m_{\!\scriptscriptstyle \psi}: \nabla \! o \! [0; \infty)$ defined by the formula is a subaddit measure.

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